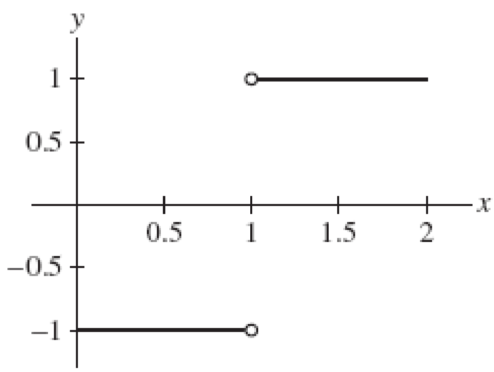
Math 1041 **Test 1 Review – Even Answer Key** **Summer 2009**

Section 2.2

38.

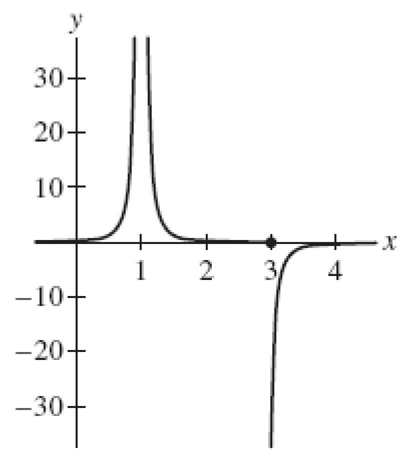
- At $c = 1$, the left-hand limit is $\lim_{t\to 1-} g(t) = 2.5$, whereas the right-hand limit is $\lim_{t\to 1+} g(t) = 0.7$. Accordingly, the two-sided limit does not exist at $c = 1$.
- At $c = 2$, the left-hand limit is $\lim_{t\to 2-} g(t) = 2$, whereas the right-hand limit is $\lim_{t\to 2+} g(t) = 3$. Accordingly, the two-sided limit does not exist at $c = 2$.
- At $c = 4$, the left-hand limit is $\lim_{t\to 4-} g(t) = 2$, whereas the right-hand limit is $\lim_{t\to 4+} g(t) = 2$. Accordingly, the two-sided limit exists at $c = 4$ and equals 2.
- At $c = 5$, the left hand limit is $\lim_{t\to 5-} g(t) \approx 1.3$, and the right-hand limit is $\lim_{t\to 5+} g(t) \approx 1.3$. Accordingly, the two-sided limit exists at $c = 5$ and is equal to roughly 1.3. Note that $g(5) \approx 1.3$ as well.

40.



$\lim_{x\to 1+} f(x) = 1$ and $\lim_{x\to 1-} f(x) = -1$

48.



Section 2.3

24.

(a) Apply the Product Law:

$$\lim_{x\to 6} f(x)^2 = \left(\lim_{x\to 6} f(x)\right)\left(\lim_{x\to 6} f(x)\right) = (4)(4) = 16.$$

(b) Since $\lim_{x\to 6} f(x) \neq 0$, we may apply the Quotient Law:

$$\lim_{x\to 6} \frac{1}{f(x)} = \frac{1}{\lim_{x\to 6} f(x)} = \frac{1}{4}.$$

(c) Apply the Product Law:

$$\lim_{x\to 6} x f(x) = \left(\lim_{x\to 6} x\right)\left(\lim_{x\to 6} f(x)\right) = 6(4) = 24.$$

Section 2.4

2.

- The function f is discontinuous at $x = 1$; it is left-continuous there.
- The function f is discontinuous at $x = 3$; it is neither left-continuous nor right-continuous there.
- The function f is discontinuous at $x = 5$; it is right-continuous there.

6. On $(-\infty, 1)$, $(1, 2)$ and $(2, \infty)$, f is defined by polynomials and so f is continuous for $x \neq 1, 2$.
At $x = 1$, $f(x)$ has a jump discontinuity because the one-sided limits exist but are not equal:

$$\lim_{x\to 1-} f(x) = \lim_{x\to 1-} (x^2 + 3) = 4, \qquad \lim_{x\to 1+} f(x) = \lim_{x\to 1+} (10 - x) = 9.$$

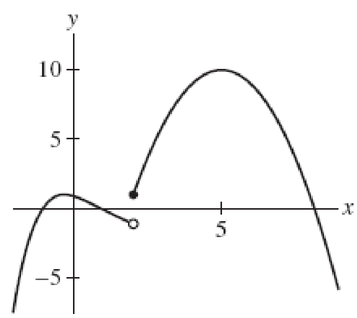
Furthermore, the right-hand limit equals the function value $f(1) = 9$, so $f(x)$ is right-continuous at $x = 1$. At $x = 2$,

$$\lim_{x\to 2-} f(x) = \lim_{x\to 2-} (10 - x) = 8, \qquad \lim_{x\to 2+} f(x) = \lim_{x\to 2+} (6x - x^2) = 8.$$

The left- and right-hand limits exist and are equal to $f(2)$, so $f(x)$ is continuous at $x = 2$.

20. For $x > 2$, $f(x) = \dfrac{x-2}{(x-2)} = 1$. For $x < 2$, $f(x) = \dfrac{(x-2)}{(2-x)} = -1$.
So f has a jump discontinuity at $x = 2$.

68. -1 **70.** 4 **82.** Right-continuous at $x = 2$.



Section 2.5

8. 8 **20.** $1/4$

Section 2.6

6. 0 **30.** $-3/4$

Chapter 2 Review Exercises

50. On $(-\infty, -2)$, $(-2, 2)$ and $(2, \infty)$, h is defined by polynomials and so h is continuous on these intervals. Setting $b = 7$ makes h continuous at $x = 2$. Since
$\lim_{x\to -2+} h(x) = \lim_{x\to -2+} (x + 1) = -2 + 1 = -1$ $\lim_{x\to -2-} h(x) = \lim_{x\to -2-} (7 - x^2) = 7 - (-2)^2 = 3$
, there is a jump discontinuity at $x = -2$.

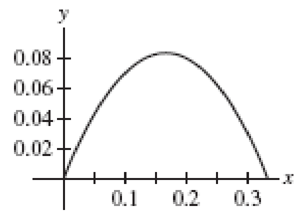
64. Let $f(x)=e^{-x^2}-x$. f is continuous on $[0,1]$ (composition and difference of continuous functions). Since $f(0)=1>0$ and $f(1)=e^{-1}-1<0$, IVT implies that there exists a $c\in(0,1)$ such that $f(c)=0$ and hence $e^{-c^2}=c$.

Section 3.1

32. $f'(0)=1/2$**;** $y=\frac{1}{2}x+1$

Section 3.2

28. $\frac{d}{dt}\left(6t^{1/2}+t^{-1/2}\right)=3t^{-1/2}-\frac{1}{2}t^{-3/2}$ **40.** $-\frac{3}{256}$ **52.** $x=1/6$



76. The function is discontinuous at $x=1$. The function is nondifferentiable at $x=1$ because it is discontinuous there and it is nondifferentiable at $x=2$ and $x=3$ because the one-sided derivatives at each of these points are not equal to each other.

Section 3.3

12. $f'(x)=\frac{(x^2+1)e^x-e^x(2x)}{(x^2+1)^2}=\frac{e^x(x-1)^2}{(x^2+1)^2}$ **50.** $F'(4)=-10$ **52.** $\frac{39}{50}$

Section 3.4

20. a) 4 cm/sec b) $t=\frac{5}{2}$ sec