Math 2043 **Test 3 Review – Even Answer Key** **Summer 2009**

Section 12.1

40. $\left\langle -2\sqrt{2}, -2\sqrt{2} \right\rangle$

Section 12.2

10. $Q = (5, 5, 4)$

27 Extra $x = 1 + 2t,\ y = 2 + t,\ z = -8 + 3t$

31 Extra $x = 1 + 2t,\ y = 1 - 6t,\ z = 1 + t$

Section 12.4

34. $\mathbf{u}_1 = \left\langle -\frac{1}{\sqrt{66}}, -\frac{4}{\sqrt{66}}, \frac{7}{\sqrt{66}} \right\rangle,\ \mathbf{u}_2 = \left\langle \frac{1}{\sqrt{66}}, \frac{4}{\sqrt{66}}, -\frac{7}{\sqrt{66}} \right\rangle$

40. 72

Section 12.5

26. $4x - 8y + 4z = 0$ or $x - 2y + z = 0$

Section 13.2

34. $\left\langle -2t, \frac{\sqrt{2}}{2} - \frac{3\sqrt{2}}{2}t, -4t \right\rangle$

42. $\left\langle \frac{1}{2}\left(1 - e^{-1}\right), -\frac{1}{2} + \ln 2 \right\rangle$

Section 13.3

4. $\frac{1}{27}\left(176^{3/2} - 32^{3/2}\right)$

19 Extra $\sqrt{3}$

Section 14.3

4. $\frac{2u + v}{u^2 + uv}$

14. $\frac{\partial z}{\partial x} = 4x^3 y + y^{-2}$ **;** $\frac{\partial z}{\partial y} = x^4 - 2xy^{-3}$

28. $\frac{\partial R}{\partial v} = -\frac{2v}{k} e^{-v^2/k}$ **;** $\frac{\partial R}{\partial k} = \frac{v^2}{k^2} e^{-v^2/k}$

42. $\ln 3 + \frac{1}{3}$

68. $R_{uvw} = \frac{2}{(v + w)^3}$

Section 14.5

6. $$\nabla g = \left\langle \frac{\partial g}{\partial x}, \frac{\partial g}{\partial y} \right\rangle = \left\langle \frac{y^2 - x^2}{\left(x^2 + y^2\right)^2}, \frac{-2xy}{\left(x^2 + y^2\right)^2} \right\rangle = \frac{1}{\left(x^2 + y^2\right)^2} \left\langle y^2 - x^2, -2xy \right\rangle$$

18. 6

27. Extra The unit vector in the direction of fastest increase is $\langle 1, 0 \rangle$**.** The rate of fastest increase is 2.

29. Extra The unit vector in the direction of fastest increase is $\left\langle \frac{1}{\sqrt{5}}, 0, \frac{-2}{\sqrt{5}} \right\rangle$**.** The rate of fastest increase is $\sqrt{5}$.

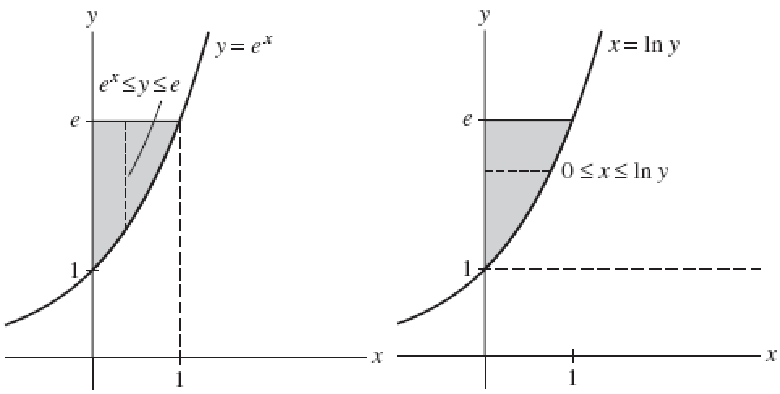
Section 15.2

4. Vertically simple region $0 \le x \le 1, \quad 0 \le y \le 1 - x^2$

Horizontally simple region: $0 \le y \le 1, \quad 0 \le x \le \sqrt{1 - y}$

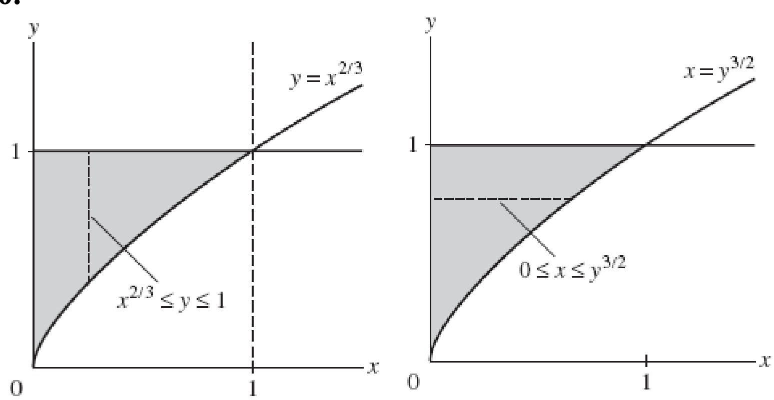
1/12

34.



$$\int_0^1 \int_{e^x}^{e} f(x, y)\, dy\, dx = \int_1^e \int_0^{\ln y} f(x, y)\, dx\, dy$$

40.

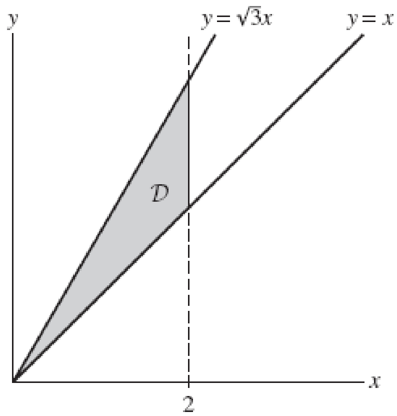


$\frac{1}{8}(e-1)$

Section 15.4

18.

$\mathcal{D} : 0 \le x \le 2,\ x \le y \le \sqrt{3}x$



$\frac{8}{3}\left(\sqrt{3}-1\right)$

34.

The inequalities describing $\mathcal{W}$ in cylindrical coordinates are thus

$$\mathcal{W} : 0 \le \theta \le 2\pi,\ 0 \le r \le 2,\ r^2 \le z \le 8 - r^2$$

The function in cylindrical coordinates is

$$f(x, y, z) = z\sqrt{x^2 + y^2} = zr \qquad \frac{1024}{15}\pi$$

Extra: $\int_0^{\pi}\int_0^{2}\int_0^{\pi/2} \rho^4 \cos\phi \sin\phi \, d\phi \, d\rho \, d\theta = \frac{16\pi}{5}$ **Extra:** $\frac{2\pi\left(1-e^{-8}\right)\left(2-\sqrt{2}\right)}{6}$

Section 16.2

48. Work = 1 along both paths.

Section 16.3

2. a) 4, b) 0, c) 0

14. A potential function is $\varphi(x, y, z) = y^2 x + e^z y$.

18. Work against **F** $= -\frac{2}{3}$ **(**work performed by **F** $= \frac{2}{3}$**) .** Along the complete square, $W = 0$

Extra (a) $\varphi(x, y) = e^{xy} - y^2 x - \cos y$ $\quad \int_C \mathbf{F} \cdot ds = e^{xy} - y^2 x - \cos y \Big|_{(1,0)}^{(0,\pi/2)} = 1$

Extra (b) $\varphi(x, y, z) = x^2 yz - \frac{z}{y} + 2z$ $\quad \int_C \mathbf{F} \cdot ds = x^2 yz - \frac{z}{y} + 2z \Big|_{(1,1,1)}^{(\sqrt{3},1,-1)} = -6$

Section 17.1

2. a) Using Green's Theorem b) -16π

4. $\frac{e^2}{2} - \frac{e^3}{3} - \frac{1}{6}$ **6.** $-\frac{1}{30}$ **10.** $\frac{\left(e^2-1\right)\left(5-e^4\right)}{2}$