

## MATH 133 PROBLEM SET 3

IGOR RIVIN, 426 WACHMAN HALL, 1X5010, RIVIN@MATH.TEMPLE.EDU

ABSTRACT. Due Friday, February 10

**Question 1.** There are 13 cards in a bridge hand. How many ways are there to arrange them left to right?

**Question 2.** Prove that for any positive integer  $n$ , the product  $n(n+1)\dots(n+9)$  is always divisible by  $10!$ .

**Question 3.** How many different bridge hands are there?

**Question 4.** Prove by induction that the sum of the first  $n$  perfect squares equals

$$\frac{n(n+1)(2n+1)}{6}$$

**Question 5.** Let  $P$  be a convex  $n$ -gon. Prove that  $P$  has

$$\frac{n(n-3)}{2}$$

diagonals.

**Question 6 – Bonus.** Prove that for any  $n$ , the *middle binomial coefficient*  $\binom{2n}{n}$  is even.

**Question 7 – Bonus.** Let  $x$  and  $y$  be integers and let  $p$  be a prime number. Prove that the remainder of  $(x+y)^p$  when divided by  $p$  equals the remainder of  $x^p + y^p$  when divided by  $p$ .

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*Key words and phrases.* discrete, mathematics, sets, logic, combinatorics, graphs.