GRADUATE MATHEMATICS COURSES, FALL 2018

Math 5043: Introduction to Numerical Analysis
MW 9:00–10:20
Prof. D. Szyld

During the first semester of this course, the student is introduced to basic concepts in numerical analysis and scientific computing. In this discipline, algorithms for the solution of specific problems arising in science and engineering using computers, are presented and analyzed. The goal is to learn algorithms which approximate the solutions, in other words, one wants to guarantee that the answer produced by the computer code resembles the true solution. At the same time, one wants to produce algorithms which converge to the solution in a reasonable amount of time.

Some of the specific methods which will be studied include: Finding roots of non-linear equations. Approximation and interpolation of functions. Numerical integration. In addition, we will study how computers store and manipulate data, so we can study how errors produced by the use of non-exact arithmetic are generated and propagated in the specific algorithms. Stability of the algorithms will be studied as well.

Textbook: J. Stoer and R. Bulirsch, Introduction to Numerical Analysis, Third Edition, Springer, 2010.

Math 8007: Introduction to Methods in Applied Mathematics I
MW 10:30–11:50
Prof. G. Queisser

This course provides the student with the toolbox of an applied mathematician: derivation of PDE, solution methods in special domains, calculus of variations, control theory, dynamical systems, asymptotic analysis, hyperbolic conservation laws.

Math 8011: Abstract Algebra I
TR 11:00–12:20
Prof. M. Lorenz

This course, the first part of a two-semester sequence, gives an introduction to the terminology and methods of modern abstract algebra. The material to be covered in the fall semester is roughly organized into three main parts: Groups (Chapters 1 – 6 in the textbook), Rings and Modules (Chapters 7 – 11), and Fields (Chapter 13). The indicated chapters contain far too much material to be completely covered in one semester; so a selection will be made. The course and its sequel (Math 8012) are prerequisites for many of the higher-level graduate courses, and together they provide the background needed for the PhD qualifying exam in Algebra.

Prerequisites: Math 3098 or equivalent or permission of instructor.

Textbook: Dummit & Foote, *Abstract Algebra*, 3rd ed., John Wiley & Sons, 2004.

Math 8041: Real Analysis I
MW 1:00–2:20
Prof. W.-S. Yang

This course covers the classical theory of Lebesgue integral and its applications. Closely related topics such as functions of bounded variation, the Riemann-Stieltjes integral, Fubini's theorem, and L^p-classes will also be discussed.

Prerequisites: Math 5041 or equivalent.

Textbook: Richard Wheeden and Antoni Zygmund, *Measure and Integral: An Introduction to Real Analysis* (Pure and Applied Mathematics) [Hardcover], CRC Press; 1st edition (1977), ISBN-10: 0824764994, ISBN-13: 978-0824764999.

Math 8051: Functions of a Complex Variable I
TR 2:30–3:50
Prof. G. Mendoza

This is the first semester of a year-long course. Topics for the first semester include: Elementary properties and examples of holomorphic functions; differentiability and analyticity, the Cauchy-Riemann equations; power series; conformality; complex line integrals, the Cauchy integral formula, Cauchy's theorem and applications; power series expansion of holomorphic functions, the Maximum Modulus Principle; Liouville's Theorem; singularities of holomorphic functions, Laurent expansions, the calculus of residues and applications to the calculation of definite integrals and sums; zeros of a holomorphic function, the Argument Principle, Rouche's Theorem, Hurwitz's Theorem; conformal mappings.

Topics for the second semester include harmonic functions, the Poisson integral formula, maximum and minimum principles, the mean value property, the Dirichlet problem, Harnack's inequality; spaces of holomorphic and meromorphic functions, the Riemann Mapping Theorem; analytic continuation; Weierstrass and Hadamard's Factorization Theorems; Picard's Theorems; introduction to Riemann Surfaces.

Prerequisites: Math 4051 or equivalent or permission of instructor.

Textbook: John B. Conway, *Functions of One Complex Variable*, Springer.

Math 8061: Differential Geometry & Topology I
TR 9:30–10:50
Prof. M. Stover

This is an introduction to the basic theory of smooth manifolds that will prepare students for that portion of the PhD qualifying exam in geometry and topology. Topics include smooth manifolds and maps, transversality, intersection theory, vectors fields, Euler characteristic, and differential forms. If there is time, we will cover additional topics like vector bundles, Morse theory, and classification of compact 1- and 2-dimensional manifolds.

Prerequisites: Concepts of Analysis (Math 5041–5042) or equivalent and Abstract Algebra (Math 8011). Abstract Algebra can be taken concurrently.

Textbook: Guillemin and Pollack, *Differential Topology*.

Math 8141: Partial Differential Equations I
TR 1:00–2:20
Prof. C. Gutierrez

A partial differential equation (PDE) is an equation that expresses a relation between a function and its partial derivatives. Since many processes (physical, chemical, etc) can be expressed in terms of rates of changes, PDEs appear and have applications to an enormous number of questions. For example, PDEs describe the propagation of sound and heat, the motion of fluids, the behavior of electric and magnetic fields, and the behavior of financial markets. PDEs are also crucial in understanding and solving various geometric problems.

The first semester course is intended to provide the student a basic introduction to the subject, including first-order PDEs and the three second order equations that arise in mathematical physics: the Laplace equation, the heat equation, and the wave equation. The solutions of these equations have different qualitative and quantitative properties and their study is essential for understanding the more general elliptic, parabolic and hyperbolic equations which will be the subject of the second semester course. The Fourier transform and Sobolev spaces and their applications to PDEs will be introduced and developed during the two semesters.

The course will be useful for students in analysis, applied mathematics, geometry, physics, and engineering.

Prerequisites: Basic concepts of real analysis; advanced calculus of several variables; knowledge of Lebesgue integration is useful.

Textbooks:

(1) L. C. Evans, *Partial Differential Equations*, Graduate Texts in Mathematics, vol. 19, American Mathematical Society, 1998, ISBN: 0-8218-0772-2.
(2) D. Gilbarg and N. S. Trudinger, *Elliptic Partial Differential Equations of Second Order*, Springer, ISBN: 9783540411604.

Math 8200: Control Theory
TR 9:30–10:50
Prof. B. Seibold

Control theory deals with the question how to affect a given system (mechanical, electric, socio-econonic, etc.) in a way that a desired system state or outcome is achieved. A fundamental challenge is to achieve this goal event in the lack of observability of the system, a lack of knowledge of the system equations, or noise. This course provides the key mathematical concepts of control theory (including observability, controllability, stabilization, open-loop and feedback control, transfer functions, and fundamentals of nonlinear and PDE-constrained control) as well as practical excercised using real robotics hardware. Control theory is a key theme in applied mathematics and engineering.

Math 9023: Knot Theory and Low-Dimensional Topology
MW 9:00–10:20
Prof. D. Futer

This course will survey the modern theory of knots, coming at it from several very distinct points of view. We will start at the beginning with projection diagrams and the tabulation problem. We will proceed to several classical polynomial invariants, which can be constructed via the combinatorics of diagrams, via representation theory, or via the topology of the knot complement. We will touch on braid groups and mapping class groups, and use these groups to show that every (closed, orientable) 3–manifold can be constructed via knots. Finally, we will use these constructions to gain a glimpse of several skein–theoretic and quantum invariants of 3–manifolds.

Prerequisites: Math 8061–62 or permission of the instructor.

Textbooks:

(1) R. Lickorish, *An introduction to knot theory*, Springer, 1997.
(2) V. Prasolov and A. Sossinsky, *Knots, links, braids, and 3–manifolds*, AMS Publications, 1997.

Math 9041: Functional Analysis
MW 10:30–11:50
Prof. Y. Grabovsky

Functional Analysis is a set of principles governing infinite dimensional linear algebra. Its range of applications is staggering from analysis of PDEs to Economics, to Quantum mechanics. This one-semester course will emphasize the fundamental ideas and the way they are applied in practice. The course style will be governed by the motto of Applied Mathematics: "seeing the abstract in the concrete". We will begin by developing the three pillars of Functional Analysis: duality, compactness and completeness. Once these foundations are built the course will cover the fundamentals of Banach and Hilbert spaces; Riesz theory of Fredholm operators; and theory of unbounded operators. Applications will include Sobolev spaces, existence and uniqueness theorems for partial differential equations, homogenization theorems and other exciting and important stuff.

Prerequisites: linear algebra, advanced calculus, and real analysis.

Math 9073: Geometric Group Theory
TR 1:00–2:20
Prof. S. Taylor

This semester-long course will survey the rapidly expanding field of geometric group theory, focusing on the role played by negative curvature. We will begin with classical combinatorial techniques used to construct and study infinite discrete groups. After introducing basic concepts in coarse geometry, we will turn our attention to Gromovs notion of hyperbolic groups. In addition to studying geometric, algebraic, and algorithmic properties of these groups, we will keep an eye towards several generalizations of hyperbolicity that have recently played a large role in understanding many geometrically significant groups. As examples, we will also touch on the study of mapping class groups, outer automorphism groups of free groups, and cubical groups.

Prerequisites: Math 8061-62.

Math 9100: Associative Algebras
TR 11:00–12:20
Prof. E. Letzter

This course will study the structure and representations of finitely generated algebras over a field. Specific topics include:

- One-dimensional representations: The Nullstellensatz (over an arbitrary field)
- Finite dimensional representations, I: Wedderburn Artin Theory
- Infinite dimensional reps: Primitive ideals and Jacobson's Density Theorem
- Finite dimensional representations II: Polynomial Identities
- Primitive ideals and representations of quantum n–space

Prerequisites: Math 8011-12.