PH.D. COMPREHENSIVE EXAMINATION
COMPLEX ANALYSIS SECTION

Fall 1995

Part I. Do three (3) of these problems.

I.1. Show that if $u(x,y)+iv(x,y)$ is an analytic function with non-vanishing derivative in a region R, then, for any constants c_1 and c_2, the curves $u(x,y)=c_1$ and $v(x,y)=c_2$ are orthogonal in R (at the points of their intersection).

I.2. If $-1 < a < 1$, compute

$$\int_0^\infty \frac{x^a}{1+x^2}dx$$

using residues.

I.3. Give a conformal (i.e., biholomorphic) map of $\mathbb{C}\setminus[1,\infty)$ onto the open unit disc.

I.4. Suppose $f(z)$ is holomorphic in $\mathbb{C}\setminus\{0\}$ and satisfies

$$|f(z)| \le |z|^2 + \frac{1}{|z|^2} \qquad \text{for } z \ne 0.$$

If $f(z)$ is an odd function, what form must it have?

Part II. Do two (2) of these problems.

II.1. Suppose $f(z)$ is meromorphic in all of $\mathbb{C}$ and bounded on $\{z : |z| > R\}$ for some $R > 0$. Prove that $f(z)$ is rational.

II.2. Suppose f is analytic on a neighborhood of the closed unit disc $\overline{D}$ and one-to-one on the unit circle ∂D. Show that f is one-to-one on $\overline{D}$.

II.3. Show that there is no one-to-one analytic function which maps $A=\{z : 0<|z|<1\}$ onto $B=\{z : 1<|z|<2\}$.